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(Printed Pages 4)

19/168

B.A./B.Sc. (Part-I) Examination, 2019

MATHEMATICS

First Paper

BMG-101

(Algebra & Trigonometry)

Time : Three Hours] *[Maximum Marks : 65*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Give answer of each part in about 50 words.

$1\frac{1}{2} \times 10 = 15$

1. (i) State Raabe's test for convergence of an infinite series.

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(ii) State Equivalence relation and give an example of it.

(iii) Define permutation group.

(iv) Define Cyclic group.

(v) Define Homomorphism and Isomorphism.

(vi) Define Conjugacy relation.

(vii) Define integral domains and fields.

(viii) Define Quotient ring.

(ix) Separate $e^{e^{ax}}$ into real and imaginary parts.

(x) Define Gregory's series.

Section-B

(Short Answer Type Questions)

Note : Attempt **all** questions. Give answer of each question in about 200 words. $6 \times 5 = 30$

2. If $<S_n>$ is a sequence such that $S_n > 0$ for all n and $\lim S_n = \ell$ then prove that

$$\lim (S_1 S_2 \ldots S_n)^{\frac{1}{n}} = \ell.$$

OR

Test for convergence the series :

$$\frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \ldots\ldots\ldots$$

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3. Define an equivalence relation. If R is a relation in the natural numbers N such that:

$R=\{(x, y) : x, y \in N$ and $x-y$ is divisible by $7\}$.

Prove that R is an equivalence relation.

OR

Show that the set of matrices

$$A_\alpha=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix} \text{ Where } \alpha\in R,$$

form a group under matrix multiplication.

4. A subgroup H of a group G is normal if and only if $x H x^{-1} = H \ \forall \ x \in G$. Prove it.

OR

Let G be a group and H is a subgroup of G ; f an automorphism of G such that $f(H) = \{f(h) : h\in H\}$ then prove that f(H) is a subgroup of G.

5. If R is a ring, then for all $a, b \in R$ prove that $a(-b)=-(ab)=(-a)b$.

OR

Show that the set of numbers of the form $a+b\sqrt{2}$ Where a and b are rational numbers is a field.

6. Resolve $e^{\cosh(x+iy)}$ into real and imaginary parts.

OR

Sum the series :

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta)+\ldots\ldots$ to n terms.

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Give answer of each question in about 500 words.

$10 \times 2 = 20$

7. Test for convergence the series:

$$1+\frac{a.b}{c}\frac{z}{\lfloor 1}+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{\lfloor 2}+\frac{a(a+1)(a+2)b(b+1)(b+2)}{c(c+1)(c+2)}\frac{z^3}{\lfloor 3}+\ldots\ldots\ldots\infty$$

8. Prove that every cyclic group is an abelian group.

9. State and prove Cayley's theorem.

10. Show that the set of matrices $\begin{bmatrix}a & b \\ o & c\end{bmatrix}$ is a subring of the ring of 2×2 matrices with integral elements.

11. Prove that $\cosh^{-1} z=\log[z+\sqrt{z^2-1}]$.